Technology Innovation Institute

A universal framework for holographic mimo sensing

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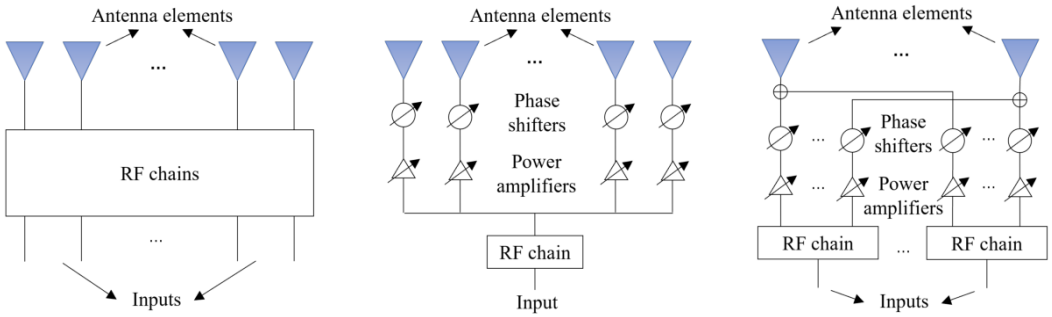
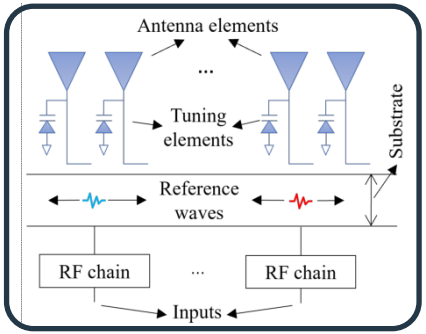
what is holographic mimo?

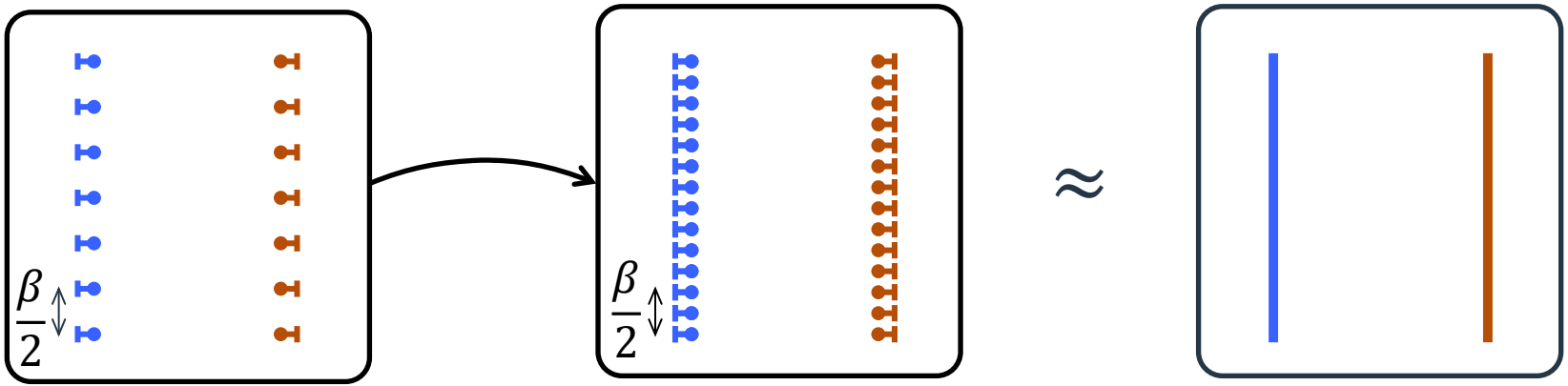
mimo systems



holographic mimo system



Gong, T. et al. Arxiv:2212.01257 (2020)

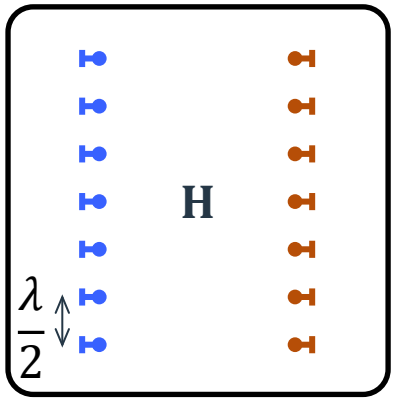


why focusing on the signal space structure of antennas?

- limits of spatial multiplexing | N, or degrees of freedom (dof)
- design of mimo precoder & combiner | ψ_i

$$u(\mathbf{r}) \approx \sum_{i=1}^{N} \gamma_i \psi_i(\mathbf{r})$$

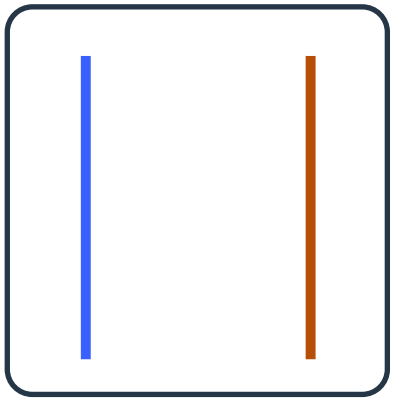
su-mimo



$N \leq$ # elements

svd($\mathbf{H}$) $\rightarrow \psi_i$

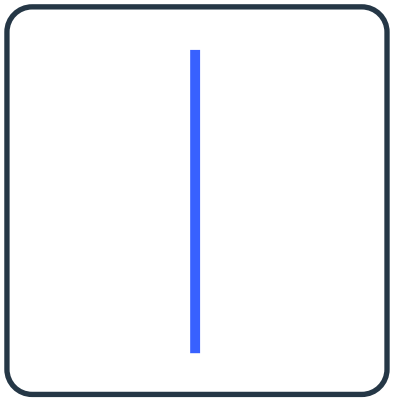
su-hmimo[1]



$N = f(\text{apertures}, \lambda)$

continuous svd $\rightarrow \psi_i$

rx hmimo?



N?

ψ_i?

[1] Dardari, D. et al. IEEE J. Sel. Areas Commun. (2020)

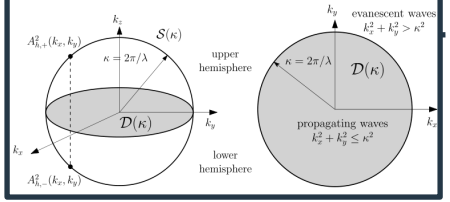
signal space basis & degrees of freedom: existing works

- analytical identities & plane waves expansions[1]

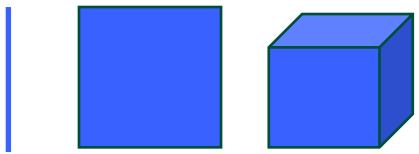
→ segment, circle & sphere ***only***

- sampling theory & Fourier plane wave expansion[2]

reparametrize in the plane

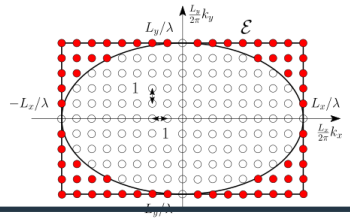


choose a ***cartesian*** shape



$L \gg \beta$

build the wavenumber lattice

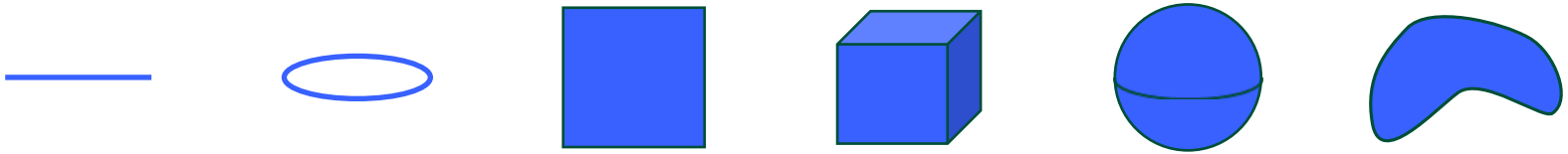


expand with Fourier plane waves

$$u(\mathbf{r}) \approx \sum_{i=1}^{N} \gamma_i \psi_i(\mathbf{r})$$

[1] Poon, A. S. Y. et al. IEEE Trans. Inform. Theory (2005)
[2] Pizzo, A. et al. IEEE J. Sel. Areas Commun. (2020)

examples



$N =$	$\frac{2L}{\beta}$	$\frac{\pi L}{\beta}$	$\frac{\pi L^2}{\beta^2}$	$\frac{2\pi L^2}{\beta^2}$	$\frac{\pi^2 L^2}{\beta^2}$	?
$\psi_i =$	~ 1d Fourier	1d Fourier	~ 2d Fourier	~ stacked 2d Fourier	spherical harmonics	?

[1] Poon, A. S. Y. et al. IEEE Trans. Inform. Theory (2005)
[2] Pizzo, A. et al. IEEE J. Sel. Areas Commun. (2020)

scalar Helmholtz equation

3d wave equation

$$\nabla^2 u(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}u(\mathbf{r},t) = 0$$

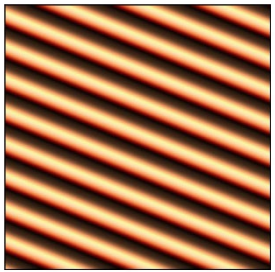
$\nabla^2 = \sum_{n=1}^{3} \partial^2/\partial r_n^2$

at one wavelength β

$$\nabla^2 u(\mathbf{r}) + \kappa^2 u(\mathbf{r}) = 0$$

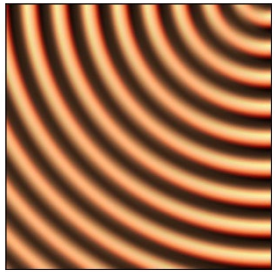
$\kappa = 2\pi/\beta$

plane waves



$u(\mathbf{r}) = e^{J\kappa \mathbf{d}(\Theta).\mathbf{r}}$

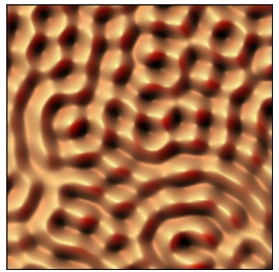
spherical waves



$u(\mathbf{r}) = \frac{e^{J\kappa\|\mathbf{r}-\mathbf{r}_0\|}}{4\pi\|\mathbf{r}-\mathbf{r}_0\|}$

cylindrical waves

general solutions



our method

going through a “universal” framework to obtain the dof and basis:

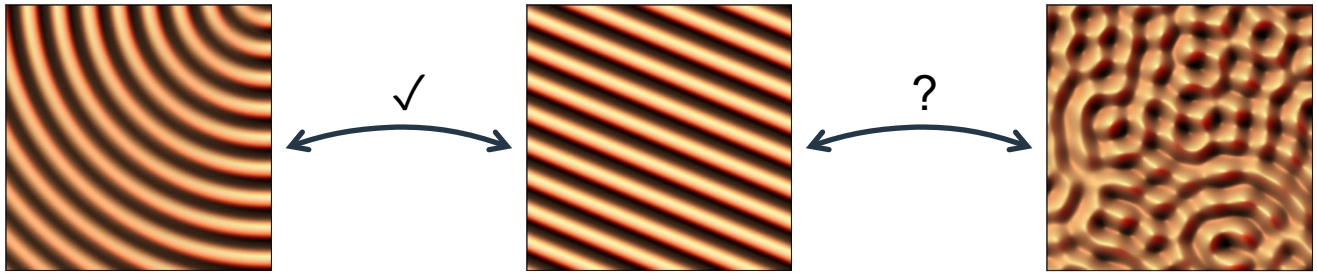
1. any antenna of finite shape → ***conformal antennas***
2. minimal conditions on what the antenna measures

💡

#1 go numerical

#2 leverage the signal structure in the spatial & wavenumber domains

#3 look at the interplay between plane waves and general solutions

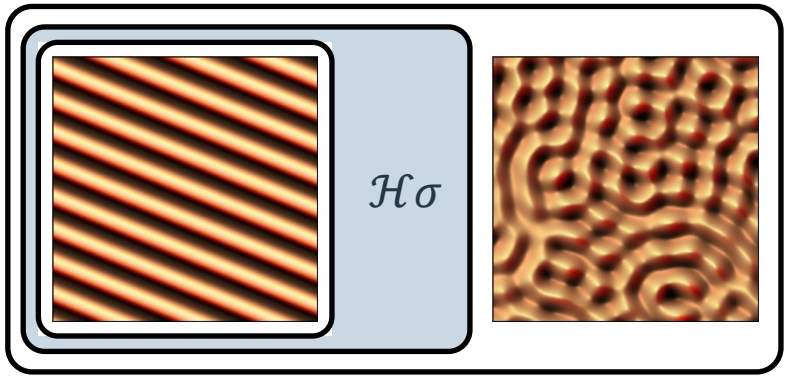


Herglotz wave functions

assume $\sigma \in \mathbb{C}$, square integrable on the sphere. Then

$$u(\mathbf{r}) = \mathcal{H}\sigma = \iint_{4\pi} \sigma(\boldsymbol{\Theta}) e^{\jmath\kappa \mathbf{d}(\boldsymbol{\Theta}).\mathbf{r}} \mathrm{d}\boldsymbol{\Theta}$$

- abuse of notation: Lebesgue to Riemann integral
- $\mathcal{H}\sigma$ are Helmholtz solutions



from Helmholtz solutions to plane waves: *finite* apertures i

with the H_1 Sobolev norm $\|u(\mathbf{r})\|_{H_1}^2 = \iiint_\Omega |u(\mathbf{r})|^2 + \sum_{n=1}^{3} \left|\frac{\partial u(\mathbf{r})}{\partial r_n}\right|^2 \mathrm{d}\mathbf{r}$

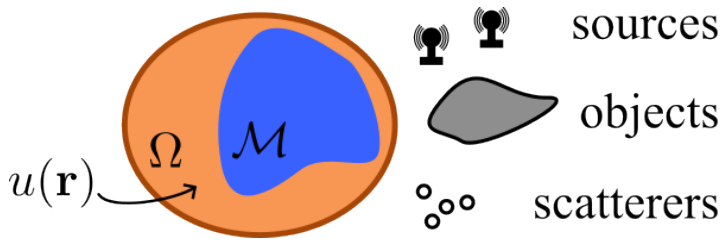
we have

Theorem[1]**:** *the set of Herglotz wave functions is dense in the space of Helmholtz solutions*

choose Ω star-convex, set $\epsilon > 0$, then

$$\exists\ \sigma \text{ such that } \|u(\mathbf{r}) - \mathcal{H}\sigma\|_{H_1}^2 < \epsilon$$

- no condition ***outside*** Ω!
- ***trick***: find Ω star-convex such that $\mathcal{M} \subset \Omega$



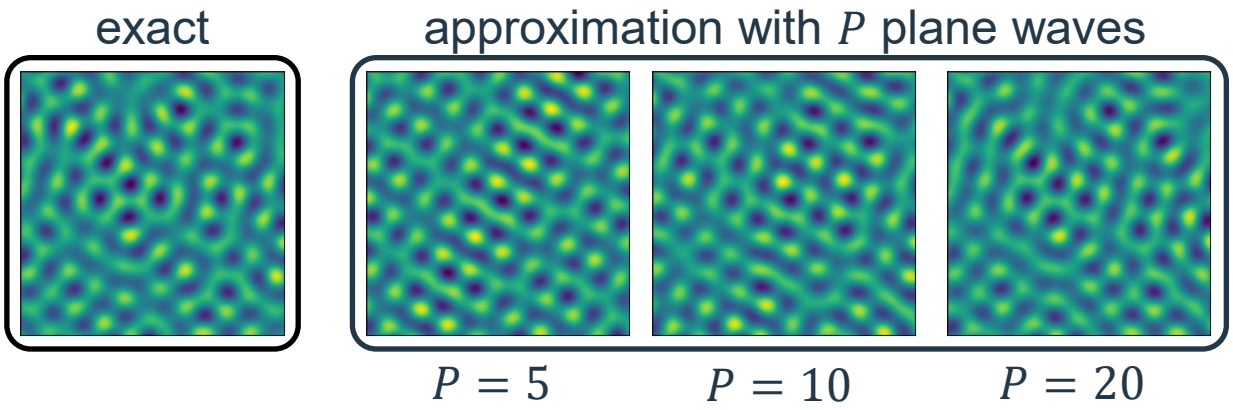
[1] Colton, D. et al. Proceedings of the Edinburgh Mathematical Society (2001)

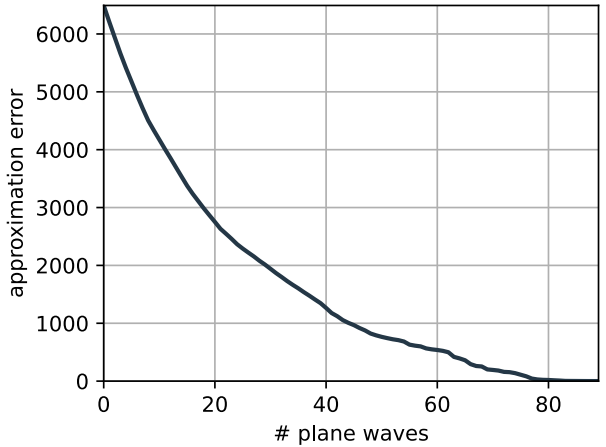
from Helmholtz solutions to plane waves: *finite* apertures ii

- approximation with a sum of plane waves? Exponential decay was proven![1]

$$u(\mathbf{r}) \approx \sum_{n=1}^{P} \alpha_n e^{\jmath \kappa \mathbf{d}(\Theta_n).\mathbf{r}}$$

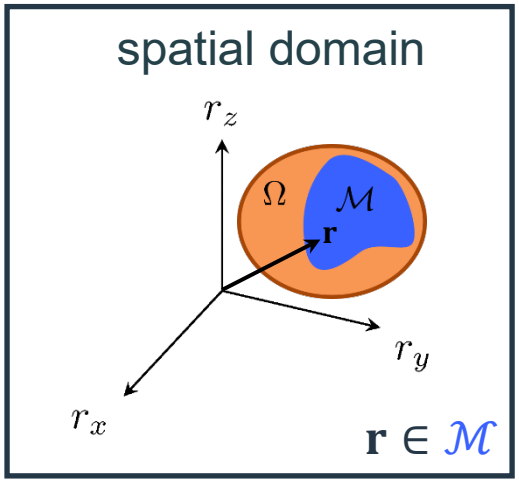
- Ω also star-convex

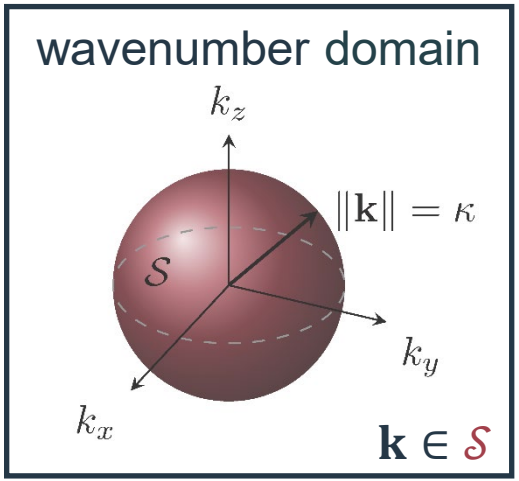


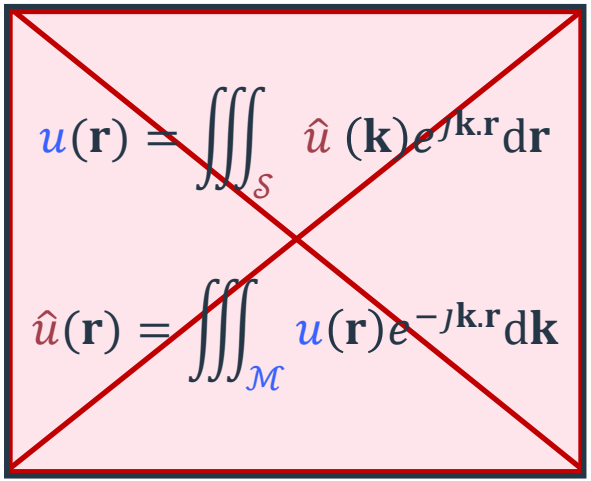


[1] Moiola, A. et al. Zeitschrift für angewandte Mathematik und Physik (2011)

leveraging the signal structure by Slepian concentration[1]





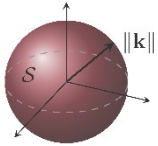


- u cannot be limited in both spaces!
- rather find functions ψ_i such that

ψ limited exactly in $\mathcal{M}$ $\hat{\psi}$ maximally concentrated in $\mathcal{S}$	or	$\hat{\psi}$ limited exactly in $\mathcal{S}$ ψ maximally concentrated in $\mathcal{M}$

[1] Slepian, D. Bell System Technical Journal (1964)

Slepian concentration for 3-d wave sensing



ψ limited exactly in $\mathcal{M}$
$\hat{\psi}$ maximally concentrated in $\mathcal{S}$

$$\lambda = \max_{\hat{\psi}=\mathcal{F}\psi} \frac{\iiint_{\mathcal{S}} |\hat{\psi}(\mathbf{k})|^2 \mathrm{d}\mathbf{k}}{\iiint_{\mathbb{R}^3} |\hat{\psi}(\mathbf{k})|^2 \mathrm{d}\mathbf{k}}$$
$$\text{s.t. } \psi(\mathbf{r}) = 0 \text{ for } \mathbf{r} \in \mathbb{R}^3 \backslash \mathcal{M}$$

$$\sim \max_{\boldsymbol{v}} \frac{\boldsymbol{v}^T \boldsymbol{A} \boldsymbol{v}}{\boldsymbol{v}^T \boldsymbol{v}}$$

$$\iiint_{\mathcal{M}} K(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}')\mathrm{d}\mathbf{r}' = \lambda\psi(\mathbf{r})$$
$$\mathbf{r} \in \mathcal{M},\ K(\mathbf{r},\mathbf{r}') = \frac{\iint_{\mathcal{S}} e^{\jmath \mathbf{k}.(\mathbf{r}-\mathbf{r}')} \mathrm{d}\mathbf{k}}{(2\pi)^3}$$

$$\sim \boldsymbol{A}\boldsymbol{v} = \lambda \boldsymbol{v}$$

$$K(\mathbf{r},\mathbf{r}') = \frac{2}{\beta^2} \mathrm{sinc}(\kappa \|\mathbf{r}-\mathbf{r}'\|)$$

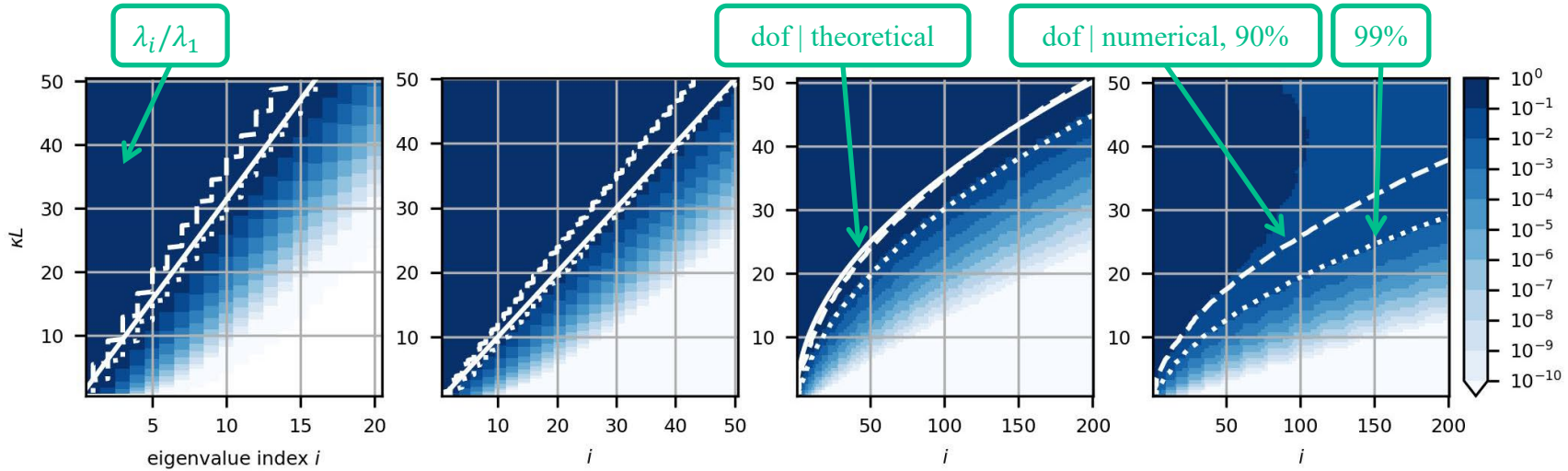
$\hat{\psi}$ limited exactly in $\mathcal{S}$
ψ maximally concentrated in $\mathcal{M}$

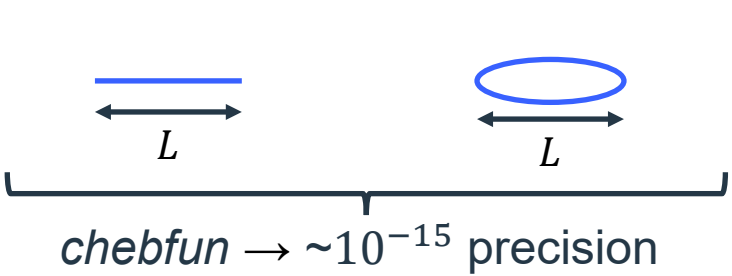
$$\iiint_{\mathcal{S}} K'(\mathbf{k},\mathbf{k}')\hat{\psi}(\mathbf{k}')\mathrm{d}\mathbf{k}' = \lambda\hat{\psi}(\mathbf{k})$$
$$\mathbf{k} \in \mathcal{S},\ K'(\mathbf{k},\mathbf{k}') = \frac{\iint_{\mathcal{S}} e^{\jmath (\mathbf{k}-\mathbf{k}').\mathbf{r}} \mathrm{d}\mathbf{k}}{(2\pi)^3}$$

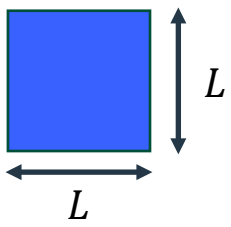
$K'(\mathbf{k},\mathbf{k}')$?

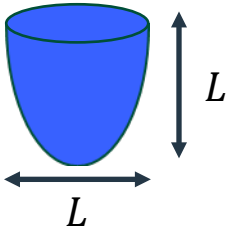
results i - numerical analysis of 4 shapes: λ_i's, dofs

numerical dof: $\sum_{i=1}^{\mathrm{DoF}} \lambda_i$ counting for 90 (99%) of $\sum_{i=1}^{\infty} \lambda_i$



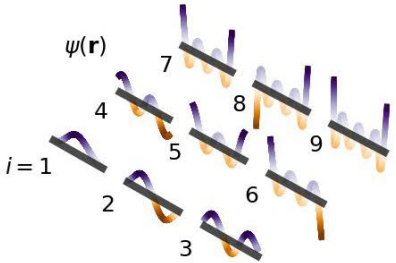


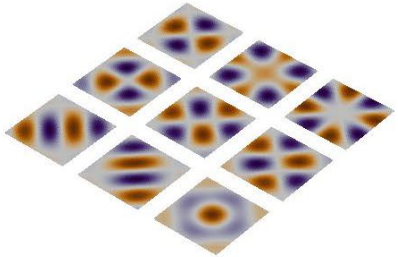




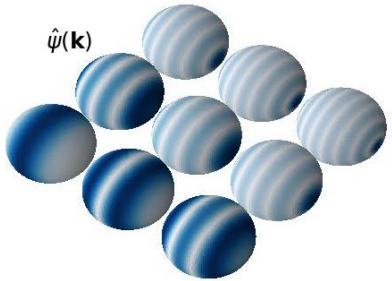
results ii - numerical analysis of 4 shapes: basis ψ_i's

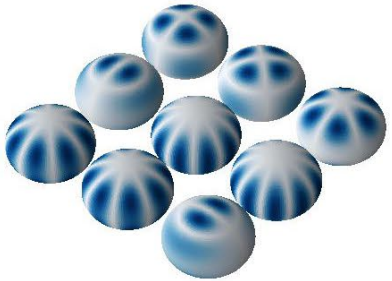
- line = time & band-limited functions → prolate spheroidal wave functions!
- circle: "circulant" kernel K → Fourier basis

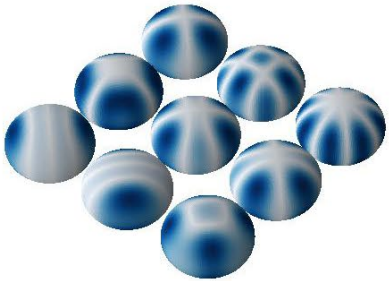


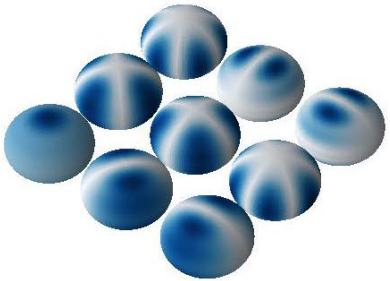


- $\hat{\psi}_i$ are orthogonal ***on the sphere*** $\mathcal{S}$ → $|\hat{\psi}_i|$ are "directivity patterns" of ψ_i



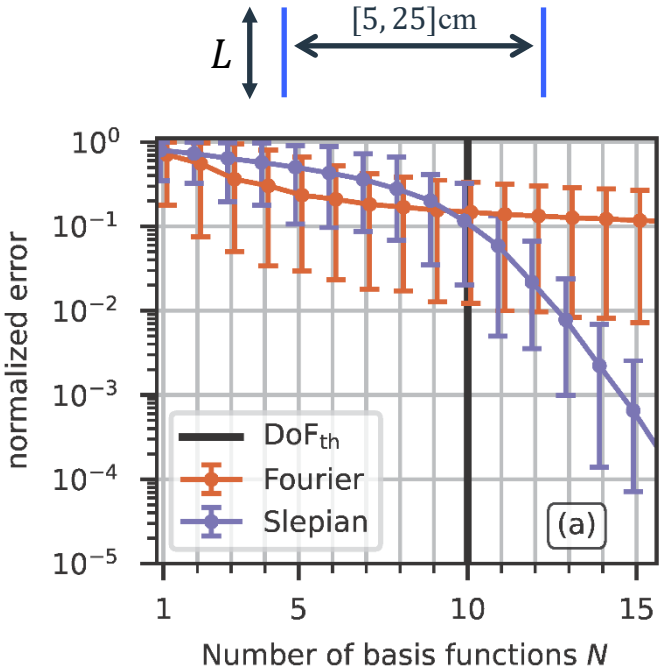


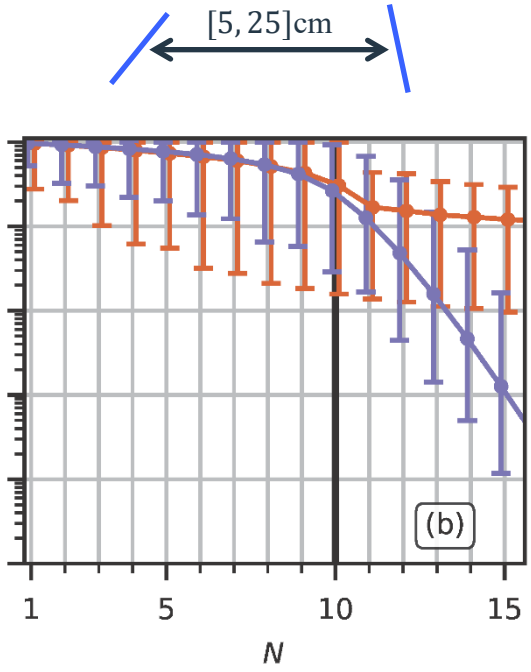




results iii - comparing with the Fourier plane wave expansion

- how accurate is the representation $u(\mathbf{r}) \approx \sum_{i=1}^{N} \gamma_i \psi_i(\mathbf{r})$?
- ψ_i : compare Slepian and Fourier | N: compare w.r.t. dof





$\beta = 1\mathrm{cm}$
$L = 5\mathrm{cm}$

conclusions & future works

what is done

- “universal” framework to analyze holographic antennas
- computational approach, rather than theoretical
- arbitrary shape

to do next…

- polarization
- mutual coupling
- holographic mimo channel modeling between arbitrary shapes

Technology Innovation Institute

thank you

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to contact me →