Walsh meets OAM in Holographic MIMO

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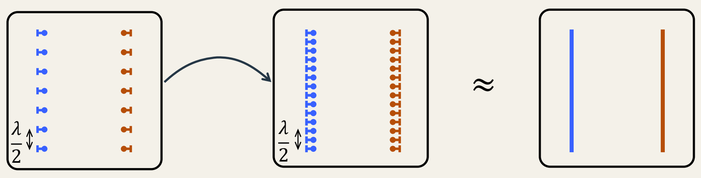
What is Holographic MIMO?



$\psi = \boldsymbol{G}\boldsymbol{\phi}$ $\quad \psi(\boldsymbol{r}) = \int_{\mathcal{S}_t} G(\boldsymbol{r},\boldsymbol{r}')\,\phi(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}'$

Physical properties

- Aperture is large
- Antenna elements are densely packed → nearly continuous
- Continuous aperture case gives upper-bound performance

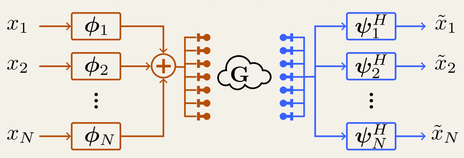
Hardware properties

- LIS are reconfigurable & tunable
- Low-cost manufacturing
- Low power consumption → high energy efficiency

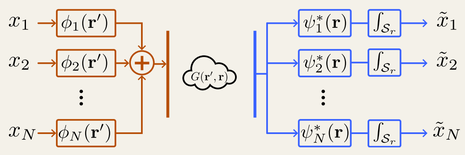
Practical consequences

- Near-field capability of communication & positioning
- MIMO performance scales with the number of elements
- H-MIMO performance scales with the LIS size

Full MIMO architecture



Full H-MIMO architecture



LIS: large intelligent surfaces | MIMO: multi-input multi-output | H-MIMO: holographic MIMO

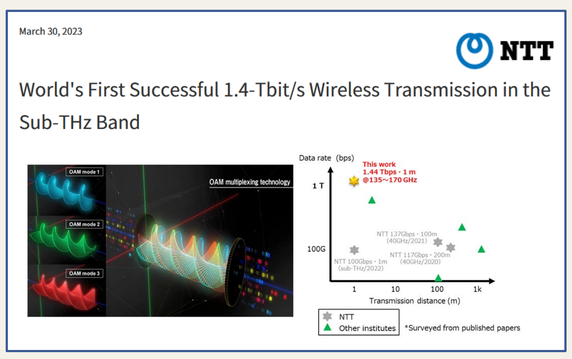
Leveraging OAM in MIMO & H-MIMO

Why OAM in MIMO

- Hypothesis: two aligned UCAs
- $\boldsymbol{G}$ is **circulant** in both near & far field

→ Optimal modes $\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \ldots \boldsymbol{\phi}_N$ & $\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \ldots \boldsymbol{\psi}_N$ are **Fourier basis**

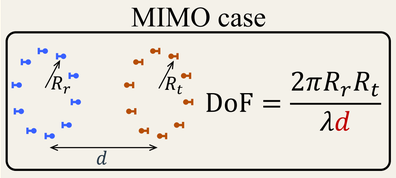
→ Convenient hardware implementation up to THz



⚠ performance of OAM ≤ performance of MIMO

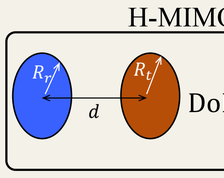
Why OAM in H-MIMO

MIMO case

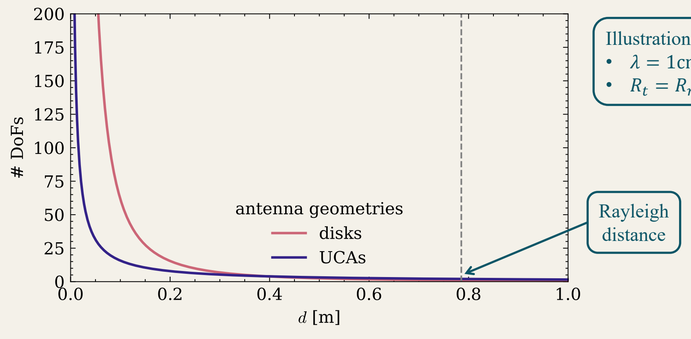


$\mathrm{DoF} = \frac{2\pi R_r R_t}{\lambda d}$

H-MIMO case



$\mathrm{DoF} = \frac{(\pi R_r R_t)^2}{\lambda^2 d^2}$



More DoF → higher near-field capability

⚠ Optimal modes are not hardware-friendly to implement

OAM: orbital angular momentum | UCA: uniform circular array | DoF: degrees of freedom

Objectives

- Design H-MIMO communication modes:
  1. for disk-shaped antennas
  2. with hardware-friendly precoding stage
- Compare with optimal case & existing solutions
- Analyze capacity in near-field line of sight scenario

Signal model & optimal modes

Continuous model

$$\psi(\boldsymbol{r}) = \int_{\mathcal{S}_t} G(\boldsymbol{r},\boldsymbol{r}')\,\phi(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}'$$

LoS propagation

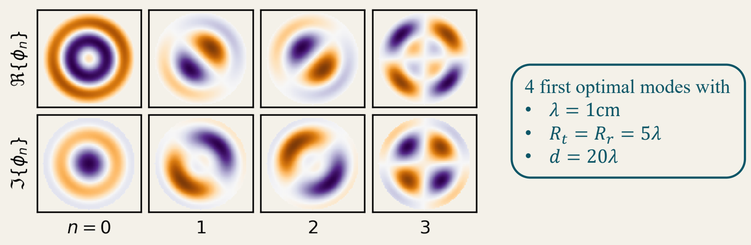
$$G(\boldsymbol{r},\boldsymbol{r}') = \frac{e^{\jmath\kappa\|\boldsymbol{r}-\boldsymbol{r}'\|}}{4\pi\|\boldsymbol{r}-\boldsymbol{r}'\|}$$

Wavenumber

Optimal Tx communication modes

$$\int_{\mathcal{S}_t} K_t(\boldsymbol{r},\boldsymbol{r}')\phi(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}' = \nu\phi(\boldsymbol{r})$$

Kernel = $\int_{\mathcal{S}_r} G^*(\boldsymbol{r},\boldsymbol{s})G(\boldsymbol{r}',\boldsymbol{s})\mathrm{d}\boldsymbol{s}$ | Eigenvalues | Optimal modes: eigenfunctions



Optimal ϕ_n admit a corresponding analytical solution:

$$\phi_{m,l}(\boldsymbol{r}) = \frac{1}{\sqrt{2\pi R_t\|\boldsymbol{r}\|}} f_{m,l}\left(\frac{\|\boldsymbol{r}\|}{R_t}\right) e^{\jmath\left(\kappa\frac{\|\boldsymbol{r}\|^2}{2d} + l\theta\right)}$$

Circular prolate spheroidal functions

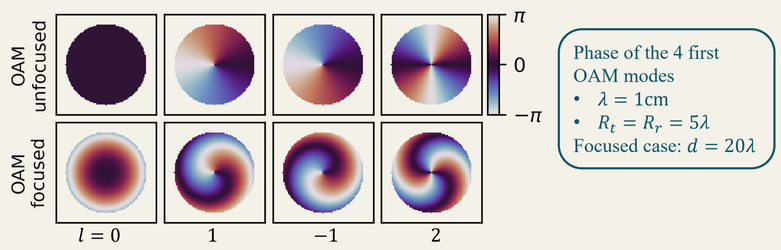
Separation of amplitude & phase terms

LoS: line of sight

OAM modes

From the basis $\phi_{m,l}$: **keep phase, change amplitude to constant**

- Spiral pattern focuses to the Rx disk
- Choosing $d = \infty$ leads to conventional OAM modes

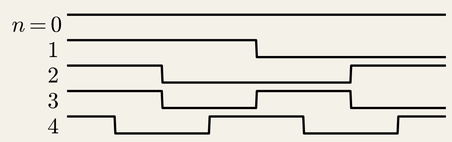


Precoding stage simplification: phase shifters

How to simplify more?

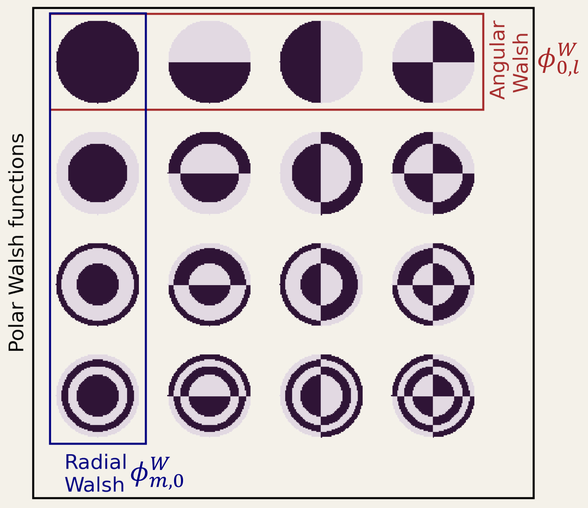
Polar Walsh functions

Walsh functions: binary-valued basis on the segment $[0, 1]$



Polar Walsh functions: binary-valued basis on the disk

$$\phi^W_{m,l}(\boldsymbol{r}) = \frac{1}{\sqrt{\pi}R_t} w_m\left(\left(\frac{\|\boldsymbol{r}\|}{R_t}\right)^2\right).w_l\left(\frac{\theta}{2\pi}\right)$$



Angular Walsh functions $\phi^W_{0,l}$ = binarized OAM unfocused modes

Precoding stage simplification: binary phase shifters

Capacity analysis

Numerical analysis of Tx-aware capacity

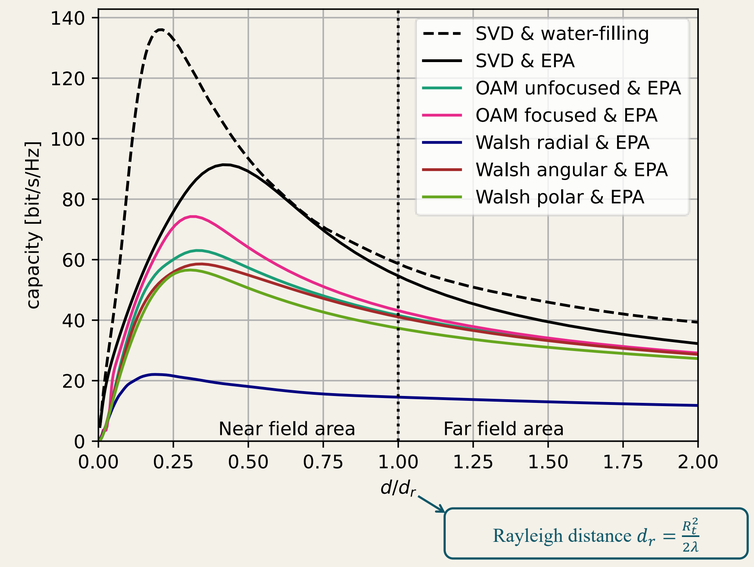
- Capacity in matrix form
- Tx & Rx disk antennas discretized x-wise & y-wise by $\frac{\lambda}{8}$

$$C = \log_2\det\left(\boldsymbol{I}_{N_r} + \frac{P_t}{\sigma^2}\boldsymbol{G}\boldsymbol{\Phi}\boldsymbol{Q}\boldsymbol{\Phi}^H\boldsymbol{G}^H\right)$$

Constant received SNR $P_t = \mathrm{SNR}.(4\pi d\sigma)^2$ | Number of Tx elements | Noise variance | EPA: $\boldsymbol{Q} = \frac{\boldsymbol{I}_N}{N}$

Results for $N = 16$ modes, SNR $= -20$dB

SVD: optimal modes	ϕ_n	$n \in [\![0, 15]\!]$
OAM focused & unfocused	$\frac{1}{\sqrt{\pi R_t}} e^{\jmath\left(\kappa\frac{\|\boldsymbol{r}\|^2}{2d} + l\theta\right)}$	$l \in [\![-8, 7]\!]$
Angular Walsh functions	$\phi^W_{0,l}$	$l \in [\![-8, 7]\!]$
Radial Walsh functions	$\phi^W_{m,0}$	$m \in [\![-8, 7]\!]$
Polar Walsh functions	$\phi^W_{m,l}$	$m, l \in [\![-2, 1]\!]$



EPA: equal power allocation

Conclusions

- Maximum capacity in the near-field area
- Comparison of capacities with different precoders:
  - non-optimal at least twice smaller than optimal
  - polar & angular Walsh ≈ OAM unfocused

→ **Binary quantization of phase shifters has little impact**

- With $N \leq 25$: same relative trends, but absolute values change

Future works

- Capacity analysis with full chain: decoder & combiner
- Impact for an actual hardware implementation?
- Impact of hardware impairments & antenna misalignment?
- Optimal transmit power allocation strategy: N & $\boldsymbol{Q}$?

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